

ТИПОВОЙ РАСЧЕТ
«Дифференциальное исчисление
функции одной действительной переменной»

Задание 1. Найти производную функции.

1.1. $y = (2x + 2)^7 + \frac{1}{\sqrt[7]{x^2}} + \sqrt{8x} .$

1.2. $y = (2x + 5)^5 + \frac{1}{\sqrt[4]{x^3}} + \sqrt{3x} .$

1.3. $y = (7x - 5)^4 + \frac{1}{\sqrt[7]{x^2}} + \sqrt{5x} .$

1.4. $y = (2x - 7)^6 + \frac{1}{\sqrt[6]{x^5}} + \sqrt{7x} .$

1.5. $y = (3x - 1)^7 + \frac{1}{\sqrt{x^7}} + \sqrt{2x} .$

1.6. $y = (2x - 3)^{10} + \frac{1}{\sqrt[7]{x^3}} + \sqrt{7x + 1} .$

1.7. $y = (9x + 5)^8 + \frac{1}{\sqrt[5]{x^3}} + \sqrt{15x} .$

1.8. $y = (3x + 5)^7 + \frac{1}{\sqrt[5]{x^7}} + \sqrt{13x - 1} .$

1.9. $y = (7x + 5)^{11} + \frac{1}{\sqrt[8]{x^3}} + \sqrt{17x} .$

1.10. $y = (5x - 1)^7 + \frac{1}{\sqrt{x^3}} + \sqrt{10x - 2} .$

1.11. $y = (3x + 2)^5 + \frac{1}{\sqrt[7]{x^4}} + \sqrt{11x} .$

1.12. $y = (3x - 5)^6 + \frac{1}{\sqrt[7]{x^5}} + \sqrt{13x - 3} .$

$$\mathbf{1.13.} \quad y = (7x+2)^9 + \frac{1}{\sqrt[14]{x^7}} + \sqrt{2x+5}.$$

$$\mathbf{1.14.} \quad y = (9x-5)^{10} + \frac{1}{\sqrt[13]{x^8}} + \sqrt{3x+8}.$$

$$\mathbf{1.15.} \quad y = (5+11x)^7 + \frac{4}{\sqrt[7]{x^3}} + \sqrt{5x-2}.$$

$$\mathbf{1.16.} \quad y = (3-4x)^8 + \frac{3}{\sqrt[7]{x^6}} + \sqrt{13x}.$$

$$\mathbf{1.17.} \quad y = (5-2x)^{13} + \frac{11}{x\sqrt{x^2}} + \sqrt{2x+5}.$$

$$\mathbf{1.18.} \quad y = (1-7x)^7 + \frac{5}{\sqrt[7]{x^9}} + \sqrt{2x-15}.$$

$$\mathbf{1.19.} \quad y = (1-5x)^5 + \frac{1}{\sqrt[8]{x^7}} + \sqrt{8x+2}.$$

$$\mathbf{1.20.} \quad y = (9-2x)^6 + \frac{1}{\sqrt[9]{x^8}} + \sqrt{5x+17}.$$

$$\mathbf{1.21.} \quad y = (3-5x)^7 + \frac{1}{2x\sqrt[5]{x^2}} + \sqrt{15x}.$$

$$\mathbf{1.22.} \quad y = (7-2x)^6 + \frac{10}{\sqrt[9]{x^7}} + \sqrt{3x-3}.$$

$$\mathbf{1.23.} \quad y = (3x-2)^5 + \frac{1}{\sqrt[5]{x^5}} + \sqrt{3-5x}.$$

$$\mathbf{1.24.} \quad y = (2x-10)^8 + \frac{1}{\sqrt[14]{x^3}} + \sqrt{3x+2}.$$

$$\mathbf{1.25.} \quad y = (2x-3)^7 + \frac{3}{x\sqrt[7]{x^2}} + \sqrt{2-11x}.$$

$$\mathbf{1.26.} \quad y = (5x+5)^7 + \frac{1}{\sqrt[12]{x^7}} + \sqrt{15x-3}.$$

$$1.27. \quad y = (5x+1)^5 + \frac{5}{\sqrt{x^7}} + \sqrt{11x-7}.$$

$$1.28. \quad y = (3x+3)^3 + \frac{1}{\sqrt[3]{x^7}} + \sqrt{9x}.$$

$$1.29. \quad y = (7x+2)^4 + \frac{1}{7x\sqrt[7]{x^3}} + \sqrt{10x}.$$

$$1.30. \quad y = (5x+7)^5 + \frac{7}{x^2\sqrt{x^3}} + \sqrt{11x+1}.$$

Задание 2. Найти производную функции.

$$2.1. \quad y = \ln(\sqrt{1+x} + x^2) + \ln 4.$$

$$2.2. \quad y = \ln(\sqrt{1+2x} + 5x^2) + \ln 5.$$

$$2.3. \quad y = \ln(\sqrt{2x-5} - x^3) + \ln 10.$$

$$2.4. \quad y = \ln(\sqrt{1+5x} - 3x) + \ln 7.$$

$$2.5. \quad y = \ln(\sqrt{x-7} + 3x^3) + \ln 3.$$

$$2.6. \quad y = 3 \ln(\sqrt{7x-3} + x^3) + \ln 8.$$

$$2.7. \quad y = \frac{1}{3} \ln(\sqrt{2x-3} + 7x^2) + \ln 9.$$

$$2.8. \quad y = 5 \ln(\sqrt{2x+2} + 3x^3) + \ln 11.$$

$$2.9. \quad y = \frac{1}{5} \ln(\sqrt{3x-1} + 5x) + \ln 13.$$

$$2.10. \quad y = 2 \ln(\sqrt{3x+1} + 6x^2) + \ln 15.$$

$$2.11. \quad y = \frac{1}{2} \ln(\sqrt{2-3x} - 5x^3) + \ln 12.$$

$$2.12. \quad y = \ln(\sqrt{5+3x} - 10x^2) + \ln 2.$$

$$2.13. \quad y = \ln(\sqrt{3+5x} + x^5) + \ln 6.$$

$$2.14. \quad y = \ln(\sqrt{7-x} - x^2) + \ln 14.$$

- 2.15.** $y = \ln(\sqrt{5-2x} - 3x^2) + \ln 15.$
- 2.16.** $y = \ln(\sqrt{2+2x} + 3x^2) + \ln 16.$
- 2.17.** $y = \ln(\sqrt{2x-5} - 7x^2) + \ln 3.$
- 2.18.** $y = \ln(\sqrt{2x+7} + 7x^3) + \ln 19.$
- 2.19.** $y = 7 \ln(\sqrt{7x-1} + 7x^2) + \ln 20.$
- 2.20.** $y = \frac{1}{7} \ln(\sqrt{1-7x} - x^5) + \ln 15.$
- 2.21.** $y = \ln(x^3 - \sqrt{3-3x} + 3x^3) + \ln 3.$
- 2.22.** $y = \ln(x^2 + \sqrt{3x+11}) + \ln 5.$
- 2.23.** $y = \ln(\sqrt{1-x} - x^2) + \ln 11.$
- 2.24.** $y = 9 \ln(\sqrt{2+x} + 5x^3) + \ln 10.$
- 2.25.** $y = \frac{1}{9} \ln(\sqrt{3-x} - x^5) + \ln 13.$
- 2.26.** $y = 3 \ln(\sqrt{5+2x} - x^3) + \ln 7.$
- 2.27.** $y = \frac{1}{3} \ln(\sqrt{1+9x} - 5x^2) + \ln 9.$
- 2.28.** $y = \ln(\sqrt{9x-3} + 9x^3) + \ln 5.$
- 2.29.** $y = \ln(\sqrt{5+7x} + 11x^5) + \ln 7.$
- 2.30.** $y = \ln(\sqrt{5-x} + 17x^2) + \ln 2.$

Задание 3. Найти производную функции.

- 3.1.** $y = \cos 5x \cdot 3^{9x}(x^2 + 4).$
- 3.2.** $y = \sin(3x+1) \cdot 5^{3x}(x^2 - 3).$
- 3.3.** $y = \sin 2x \cdot 2^{7x}(x^2 - 1).$
- 3.4.** $y = \cos(3x-5) \cdot 3^{3x}(x^2 - 5).$
- 3.5.** $y = \cos 7x \cdot 5^{9x}(4 - x^2).$

- 3.6. $y = \sin(5x+1) \cdot 2^{5x}(1+x^2)$.
- 3.7. $y = \cos(3x+2) \cdot 2^{9x}(1-7x^2)$.
- 3.8. $y = \cos 3x \cdot 5^{7x}(3-5x^2)$.
- 3.9. $y = \sin(2x-7) \cdot 9^{3x}(x^2+11)$.
- 3.10. $y = \sin 5x \cdot 7^{3x}(x^2+5)$.
- 3.11. $y = \sin(2-3x) \cdot 3^{5x}(2x^2+3)$.
- 3.12. $y = \cos(1-3x) \cdot 7^{5x}(3x^2-2)$.
- 3.13. $y = \cos(3x+7) \cdot 4^{7x}(5-x^2)$.
- 3.14. $y = \cos 11x \cdot 3^{7x}(11+3x^2)$.
- 3.15. $y = \sin(7x-3) \cdot 2^{7x}(x^2+9)$.
- 3.16. $y = \sin(3-5x) \cdot 2^{9x}(9-x^2)$.
- 3.17. $y = \sin 11x \cdot 2^{3x}(x^2-13)$.
- 3.18. $y = \operatorname{tg} 5x \cdot 7^{9x}(3x^2-4)$.
- 3.19. $y = \operatorname{tg} 7x \cdot 5^{9x}(4-7x^2)$.
- 3.20. $y = \operatorname{tg} 2x \cdot 9^{5x}(4-x^2)$.
- 3.21. $y = \operatorname{ctg} 7x \cdot 5^{7x}(1-5x^2)$.
- 3.22. $y = \operatorname{ctg}(3x+2) \cdot 5^{2x}(2+x^2)$.
- 3.23. $y = \operatorname{ctg}(5x-1) \cdot 11^x(3-2x^2)$.
- 3.24. $y = \operatorname{tg} 9x \cdot 4^{2x}(x^2+15)$.
- 3.25. $y = \operatorname{tg}(2x-1) \cdot 4^{3x}(7-2x^2)$.
- 3.26. $y = \operatorname{tg} 11x \cdot 7^{2x}(x^2-11)$.
- 3.27. $y = \operatorname{ctg} 2x \cdot 9^{3x}(2x^2-7)$.
- 3.28. $y = \operatorname{ctg}(2-3x) \cdot 4^{7x}(15-x^2)$.
- 3.29. $y = \operatorname{ctg}(2-3x) \cdot 4^{7x}(15-x^2)$.
- 3.30. $y = \operatorname{tg}(5x+1) \cdot 5^{2x}(3x^2-3)$.

Задание 4. Найти производную функции.

$$4.1. \quad y = \frac{x^3 + \sin x}{x^2 + 5} + e^{-2}.$$

$$4.2. \quad y = \frac{x - \cos x}{5 - x^2} + e^{-5}.$$

$$4.3. \quad y = \frac{2 \sin x - x^3}{x^2 + 3} + e^{-3}.$$

$$4.4. \quad y = \frac{x^3 - \cos x}{x^2 - 3} + e^{-4}.$$

$$4.5. \quad y = \frac{\operatorname{tg} x - x^5}{x^2 + 1} + e^{-7}.$$

$$4.6. \quad y = \frac{x^3 - \operatorname{tg} x}{1 - x^2} + e^{-6}.$$

$$4.7. \quad y = \frac{\operatorname{tg} x - 2x}{\operatorname{tg} x + 3} + e^{11}.$$

$$4.8. \quad y = \frac{x^2 + 3 \sin x}{x^2 - 5} + e^{-11}.$$

$$4.9. \quad y = \frac{x^5 - \cos 5x}{3 - x^2} + e^{-13}.$$

$$4.10. \quad y = \frac{\sin 3x - x^3}{x^5 + 1} + e^9.$$

$$4.11. \quad y = \frac{x^3 + \cos 3x}{x^3 - 1} + e^{-9}.$$

$$4.12. \quad y = \frac{x^3 + \operatorname{tg} x}{x^2 + 1} + e^{13}.$$

$$4.13. \quad y = \frac{\operatorname{tg} x - x^2}{x^5 - 3} + e^5.$$

$$4.14. \quad y = \frac{x^2 - \operatorname{ctg} x}{x^2 + 2} + e^2.$$

$$4.15. \quad y = \frac{x^4 + \operatorname{ctg} x}{x^2 - 7} + e^3.$$

$$4.16. \quad y = \frac{7 - x^2}{\sin x + \cos x} + e^4.$$

$$4.17. \quad y = \frac{x^3 - 5}{\cos x - \sin x} + e^7.$$

$$4.18. \quad y = \frac{x^5 - 5}{x^3 - \cos x} + e^{-10}.$$

$$4.19. \quad y = \frac{5 + x^4}{x^4 + \cos x} + e^{10}.$$

$$4.20. \quad y = \frac{\operatorname{tg} x + x^4}{x^2 + 11} + e^{12}.$$

$$4.21. \quad y = \frac{\sin x + \cos x}{\cos x - \sin x} + e^{-12}.$$

$$4.22. \quad y = \frac{\operatorname{tg} x + 1}{\operatorname{tg} x - 1} + e^{15}.$$

$$\text{4.23. } y = \frac{x^5 + \sin 2x}{x^2 + 2} + e^{-15}. \quad \text{4.24. } y = \frac{\cos x - \sin x}{\cos x + \sin x} + 2e^3.$$

$$\text{4.25. } y = \frac{\operatorname{tg} 3x}{x^2 + 7} + 3e^{-5}. \quad \text{4.26. } y = \frac{\operatorname{tg} 2x + x}{x^3 - 5} + 2e^{-7}.$$

$$\text{4.27. } y = \frac{x^5 + 1}{\sin x + 3} + 7e^{-3}. \quad \text{4.28. } y = \frac{\operatorname{ctg} x - 5}{x^5 + 5} + 5e^{-4}.$$

$$\text{4.29. } y = \frac{2x^7 + 1}{3 \cos x - x} + 4e^{-9}. \quad \text{4.30. } y = \frac{3x^3 + \sin 2x}{x^5 - 3} + 3e^5.$$

Задание 5. Найти производную функции.

$$\text{5.1. } y = \cos^5(2^x + x) + \operatorname{tg} \frac{1 - e^x}{1 + e^x} + \cos \ln 2.$$

$$\text{5.2. } y = \sin^6(x - 3^x) + \operatorname{tg} \frac{e^{2x}}{1 - e^x} + \sin \ln 2.$$

$$\text{5.3. } y = \cos^3(x - 2^x) + \operatorname{ctg} \frac{e^x}{1 + e^x} + \cos \ln 3.$$

$$\text{5.4. } y = \sin^5(5^x + 1) + \operatorname{ctg} \frac{1 + e^{2x}}{e^{2x}} + \operatorname{ctg} \ln 3.$$

$$\text{5.5. } y = \cos^6(1 - 5^x) + \operatorname{tg} \frac{e^x}{1 - e^x} + \cos \ln 3.$$

$$\text{5.6. } y = \cos^4(x + 5^x) + \operatorname{ctg} \frac{1 - e^x}{e^x} + \sin \ln 3.$$

$$\text{5.7. } y = \sin^3(x - 5^x) + \operatorname{tg} \frac{e^{3x}}{1 - e^x} + \operatorname{tg} \ln 5.$$

$$\text{5.8. } y = \operatorname{tg}^4(x + 3^x) + \sin \frac{1 - e^x}{1 + e^x} + \operatorname{ctg} \ln 5.$$

$$\text{5.9. } y = \operatorname{tg}^6(x + 2^x) + \cos \frac{1 - e^x}{1 + e^x} + \cos \ln 5.$$

$$5.10. \quad y = \cos^2(2^x - 3) + \sin \frac{e^{2x}}{1+e^x} + \sin \ln 5.$$

$$5.11. \quad y = \operatorname{tg}^2(3 + 2^x) - \cos \frac{1+e^{2x}}{e^{2x}} + \operatorname{tg} \ln 9.$$

$$5.12. \quad y = \operatorname{ctg}^2(3 - 5^x) - \sin \frac{e^{3x}}{1+e^{3x}} + \operatorname{ctg} \ln 9.$$

$$5.13. \quad y = \cos^3(5^x - 2) - \sin \frac{1-e^x}{e^{5x}} + \sin \ln 4.$$

$$5.14. \quad y = \sin^2(2 - 3^x) + \cos \frac{e^{4x}}{1+e^{4x}} + \cos \ln 4.$$

$$5.15. \quad y = \operatorname{tg}^5(5^x - 3x) + \sin \frac{1+e^{3x}}{e^{3x}} + \operatorname{tg} \ln 4.$$

$$5.16. \quad y = \operatorname{ctg}^4(3x - 3^x) + \cos \frac{e^{3x}}{1-e^{3x}} + \operatorname{ctg} \ln 4.$$

$$5.17. \quad y = \sin^4(5^x - x) + \operatorname{tg} \frac{e^{2x}}{1-e^{2x}} + \sin \ln 7.$$

$$5.18. \quad y = \sin^7(7^x + x) - \operatorname{ctg} \frac{1-e^{2x}}{1+e^{2x}} + \operatorname{tg} \ln 7.$$

$$5.19. \quad y = \cos^7(3^x - x) + \operatorname{tg} \frac{e^x + 1}{1-e^x} + \operatorname{ctg} \ln 7.$$

$$5.20. \quad y = \operatorname{tg}^7(7^x - x) + \sin \frac{1-e^{3x}}{1+e^{3x}} + \sin \ln 8.$$

$$5.21. \quad y = \operatorname{ctg}^7(x - 7^x) + \cos \frac{1+e^{5x}}{e^{5x}} + \cos \ln 8.$$

$$5.22. \quad y = \cos^6(x - 5^x) + \operatorname{tg} \frac{e^{4x}}{1-e^{4x}} + \operatorname{tg} \ln 8.$$

$$5.23. \quad y = \sin^8(2x - 3^x) + \operatorname{tg} \frac{e^{4x} - 1}{e^{4x} + 1} + \operatorname{ctg} \ln 8.$$

$$5.24. \quad y = \operatorname{tg}^8(3x + 2^x) + \sin \frac{1+e^{4x}}{e^{4x}} + \sin \ln 9.$$

$$5.25. \quad y = \operatorname{ctg}^8(2^x - 3x) + \cos \frac{e^{4x}}{1+e^{4x}} + \cos \ln 9.$$

$$5.26. \quad y = \cos^8(7^x + x) + \operatorname{tg} \frac{e^{5x}-1}{e^{5x}+1} + \operatorname{tg} \ln 9.$$

$$5.27. \quad y = \sin^9(3x - 2^x) - \operatorname{ctg} \frac{e^{2x}}{1-e^{2x}} + \operatorname{ctg} \ln 9.$$

$$5.28. \quad y = \cos^9(5 - 3^x) + \operatorname{tg} \frac{e^{5x}}{1-e^{5x}} + \sin \ln 10.$$

$$5.29. \quad y = \operatorname{tg}^9(7 - 7^x) + \sin \frac{1+e^{5x}}{1-e^{5x}} + \cos \ln 10.$$

$$5.30. \quad y = \operatorname{ctg}^9(3 + 3^x) + \cos \frac{1-e^{5x}}{e^{5x}} + \operatorname{tg} \ln 10.$$

Задание 6. Найти производную функции.

$$6.1. \quad y = \arccos \sqrt{1-x^3}.$$

$$6.16. \quad y = \frac{1}{3} \operatorname{tg}^3 x.$$

$$6.2. \quad y = \operatorname{ctg} \sqrt{x}.$$

$$6.17. \quad y = x^2 10^{2x}.$$

$$6.3. \quad y = 2 \cos^2(x+2).$$

$$6.18. \quad y = \operatorname{arcctg} \frac{1+x}{1-x}.$$

$$6.4. \quad y = \sqrt{1-(\arccos x)^2}.$$

$$6.19. \quad y = \ln^2 x - \ln(\ln x).$$

$$6.5. \quad y = \frac{\sin x}{2 \cos^2 x}.$$

$$6.20. \quad y = \sqrt{\ln x + 1} + \ln(\sqrt{x} + 1).$$

$$6.6. \quad y = \frac{1}{(1+\sin 4x)^3}.$$

$$6.21. \quad y = \operatorname{tg}^2 5x.$$

$$6.7. \quad y = \frac{1}{(1+\cos 2x)^3}.$$

$$6.22. \quad y = \sin^2(x^3).$$

6.8. $y = \ln\left(x + \sqrt{x^2 + 4}\right)$.

6.9. $y = \sin^4 5x$.

6.10. $y = \operatorname{arctg} x^2$.

6.11. $y = \ln \frac{1+x}{\sqrt{1+x^2}}$.

6.12. $y = \ln^2(1 - \cos x)$.

6.13. $y = \arcsin \frac{1}{\sqrt{x}}$.

6.14. $y = \operatorname{arctg} 3^{\sqrt{x}}$.

6.15. $y = \ln(\arcsin \sqrt{x})$.

6.23. $y = \arcsin(\ln x)$.

6.24. $y = \operatorname{arctg} \left(\ln \frac{1}{x} \right)$.

6.25. $y = \ln \cos \frac{x-1}{x}$.

6.26. $y = \frac{1}{\ln^2 x}$.

6.27. $y = x \sin \left(\ln x - \frac{\pi}{4} \right)$.

6.28. $y = \sqrt{\sin x^2}$.

6.29. $y = x^2 \sin \frac{1}{x}$.

6.30. $y = \frac{x}{1+e^{\frac{1}{x}}}$.

Задание 7. Найти производную функции.

7.1. $y = x^{\operatorname{tg} x}$.

7.2. $y = \sqrt[x]{x}$.

7.3. $y = \left(\frac{x}{2}\right)^{2x}$.

7.4. $y = x^{e^x}$.

7.5. $y = x^x$.

7.6. $y = (\sin x)^x$.

7.7. $y = x^{x^3}$.

7.8. $y = (\ln x)^x$.

7.16. $y = x^{\operatorname{ctgx} x}$.

7.17. $y = \sqrt[2x]{x}$.

7.18. $y = x^{2^x}$.

7.19. $y = x^{x^3}$.

7.20. $y = x^{\operatorname{arctg} x}$.

7.21. $y = (\ln x)^{3^x}$.

7.22. $y = x^{\operatorname{arctg} x}$.

7.23. $y = x^{e^{\cos x}}$.

$$7.9. \quad y = x^{\frac{1}{x}}.$$

$$7.10. \quad y = (\cos x)^{\sin x}.$$

$$7.11. \quad y = x^{\sin x}.$$

$$7.12. \quad y = (\cos x)^{\frac{1}{x}}.$$

$$7.13. \quad y = (x+1)^{\frac{1}{\sin x}}.$$

$$7.14. \quad y = x^{\arcsin x}.$$

$$7.15. \quad y = x^{\arccos x}.$$

$$7.24. \quad y = x^{\sin x^3}.$$

$$7.25. \quad y = x^{2x^2+1}.$$

$$7.26. \quad y = x^{\operatorname{ctg} x}.$$

$$7.27. \quad y = (x^4 + 1)^{\operatorname{tg} x}.$$

$$7.28. \quad y = \left(\frac{1}{x}\right)^{3x}.$$

$$7.29. \quad y = (\operatorname{tg} x)^{4e^x}.$$

$$7.30. \quad y = x^{\cos x^2}.$$

Задание 8. Найти производную функции.

$$8.1. \quad y = \log_2 \frac{\sqrt{x} + 5}{x^2 + x + 1}.$$

$$8.2. \quad y = \ln^2 \cos x.$$

$$8.3. \quad y = \sqrt{\ln(3x^2 + 2x)}.$$

$$8.4. \quad y = \arcsin \frac{x}{\sqrt{1+x^2}}.$$

$$8.5. \quad y = \ln \frac{x}{\sqrt{1-x^2}}.$$

$$8.6. \quad y = \ln^4 \frac{1}{x^2}.$$

$$8.7. \quad y = \ln \operatorname{tg} \left(\frac{\pi}{4} + \frac{x}{2} \right).$$

$$8.8. \quad y = \log_3(x^2 - 1).$$

$$8.16. \quad y = \ln \left(\sqrt{x-1} + \sqrt{x^2 + 1} \right).$$

$$8.17. \quad y = \ln \left(\arccos \frac{1}{\sqrt{x}} \right).$$

$$8.18. \quad y = \ln \left(\arccos \sqrt{1-e^{4x}} \right).$$

$$8.19. \quad y = \log_2 \frac{1}{\sqrt{1-x^4}}.$$

$$8.20. \quad y = \ln \arccos 2x.$$

$$8.21. \quad y = \ln \ln^3 \ln^2 x.$$

$$8.22. \quad y = \ln \frac{\ln x}{\sin \frac{1}{x}}.$$

$$8.23. \quad y = \log_4 \log_2 \operatorname{tg} x.$$

8.9. $y = \ln^2 \sin x.$

8.24. $y = \ln \frac{1+x}{\sqrt{1+x^2}}.$

8.10. $y = \ln^2(1 + \cos x).$

8.25. $y = \ln^2 \arcsin \sqrt[3]{x}.$

8.11. $y = \ln \frac{x^2}{1-x^2}.$

8.26. $y = \ln^2 \cos^3(4x-1).$

8.12. $y = \log_2(3x^2 + 1).$

8.27. $y = \ln \ln \ln x.$

8.13. $y = \ln \operatorname{tg} x.$

8.28. $y = \ln \sqrt{\frac{1+\operatorname{tg} x}{1-\operatorname{tg} x}}.$

8.14. $y = \left(1 + \ln \frac{1}{x}\right)^5.$

8.29. $y = \ln(x + \sqrt{x^2 - 1}).$

8.15. $y = \ln \ln \frac{1}{\sqrt{x}}.$

8.30. $y = \ln \frac{1+x^2}{1-x^2}.$

Задание 9. Найти производную $y'_x.$

9.1. $\begin{cases} x = e^t \cos t, \\ y = e^{2t} \sin t. \end{cases}$

9.16. $\begin{cases} x = a \cos^2 t, \\ y = b \sin^2 t. \end{cases}$

9.2. $\begin{cases} x = \frac{3t}{1+t^3}, \\ y = \frac{3t^2}{1+t^3}. \end{cases}$

9.17. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t) \end{cases}$

9.3. $\begin{cases} x = \cos^3 t, \\ y = \sin^3 t. \end{cases}$

9.18. $\begin{cases} x = t \ln t, \\ y = \frac{\ln t}{t}. \end{cases}$

9.4. $\begin{cases} x = \ln \operatorname{ctg} t, \\ y = \frac{1}{\cos^2 t}. \end{cases}$

9.19. $\begin{cases} x = \sqrt{1-t^2}, \\ y = \arcsin t. \end{cases}$

$$9.5. \begin{cases} x = e^{3t} \cos 2t, \\ y = e^t \sin 2t. \end{cases}$$

$$9.6. \begin{cases} x = \sqrt{1-t^2}, \\ y = \operatorname{tg} \sqrt{1+t}. \end{cases}$$

$$9.7. \begin{cases} x = \sqrt{1-t}, \\ y = \operatorname{tgt}. \end{cases}$$

$$9.8. \begin{cases} x = \ln \operatorname{tgt}, \\ y = \frac{1}{\sin^2 t}. \end{cases}$$

$$9.9. \begin{cases} x = t - \sin t, \\ y = 1 - \cos t. \end{cases}$$

$$9.10. \begin{cases} x = (\arcsin t)^2, \\ y = \frac{t}{\sqrt{1-t^2}}. \end{cases}$$

$$9.11. \begin{cases} x = \frac{1}{t+1}, \\ y = \left(\frac{t}{t+1}\right)^2. \end{cases}$$

$$9.12. \begin{cases} x = \ln \sqrt{\frac{1-t}{1+t}}, \\ y = \sqrt{1-t^2}. \end{cases}$$

$$9.13. \begin{cases} x = \sin t, \\ y = \ln \cos t. \end{cases}$$

$$9.20. \begin{cases} x = \ln \sin t, \\ y = \sqrt{1-t^2}. \end{cases}$$

$$9.21. \begin{cases} x = \ln(t^2 + 1), \\ y = \sqrt{t^2 + 1}. \end{cases}$$

$$9.22. \begin{cases} x = \operatorname{tgt} + c \operatorname{tgt}, \\ y = t \cos t. \end{cases}$$

$$9.23. \begin{cases} x = \frac{t+1}{t}, \\ y = \frac{t-1}{t}. \end{cases}$$

$$9.24. \begin{cases} x = 2(1 - \sin t), \\ y = 2(t - \cos t). \end{cases}$$

$$9.25. \begin{cases} x = 2\operatorname{tgt}, \\ y = 2\sin^2 t + \sin 2t. \end{cases}$$

$$9.26. \begin{cases} x = \cos^2 t, \\ y = \operatorname{tg}^2 t. \end{cases}$$

$$9.27. \begin{cases} x = \ln t, \\ y = \operatorname{arctgt}. \end{cases}$$

$$9.28. \begin{cases} x = \cos t, \\ y = \ln \sin t. \end{cases}$$

9.14. $\begin{cases} x = \ln(t - 2), \\ y = t + \sin t. \end{cases}$

9.15. $\begin{cases} x = \ln(4 + t^2) \\ y = \sqrt{4 + t^2}. \end{cases}$

9.29. $\begin{cases} x = \arctgt, \\ y = \frac{t^2}{2}. \end{cases}$

9.30. $\begin{cases} x = \sin t - t \cos t, \\ y = \cos t + t \sin t. \end{cases}$

Задание 10. Найти y' , y'' .

10.1. $y = \operatorname{arctgx}.$

10.16. $y = \frac{1}{x^2 - 4}.$

10.2. $y = e^{-x} \sin x.$

10.17. $y = x^2 \cos \frac{1}{x}.$

10.3. $y = (x - 2)e^{2x}.$

10.18. $y = \ln \sqrt[3]{1 + x^2}.$

10.4. $y = x^2 \ln x.$

10.19. $y = \frac{1+x}{\sqrt{x}}.$

10.5. $y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{\frac{-x}{a}} \right).$

10.20. $y = x^3 \ln x.$

10.6. $y = \sqrt{2x - x^2}.$

10.21. $y = \ln \cos x.$

10.7. $y = \cos e^x + \sin e^x.$

10.22. $y = \arcsin \frac{1}{x}.$

10.8. $y = x^2 \sin \frac{1}{x}.$

10.23. $y = x^2 \ln x.$

10.9. $y = \frac{1+x}{1-x}.$

10.24. $y = (1 + x^2)e^x.$

10.10. $y = \ln \sin x.$

10.25. $y = \operatorname{arctgx} - x.$

10.11. $y = \frac{1-x}{1+x}.$

10.26. $y = \frac{x^3}{x^2 + 12}.$

10.12. $y = \ln(1 + x^2).$

10.27. $y = \sqrt[3]{x + 2}.$

10.13. $y = \frac{x+1}{x^2+1}.$

10.14. $y = \arctg \frac{1}{x}.$

10.15. $y = \frac{1}{2} x^2 e^x.$

10.28. $y = \ln(1+x^3).$

10.29. $y = \sin^2 x.$

10.30. $y = e^{-x^2}.$

Задание 11. Составить уравнение касательной и нормали к кривым в точке x_0 .

11.1. $y = \sqrt{5+2x}, \quad x_0 = 2.$

11.2. $y = \frac{x^2}{x+1}, \quad x_0 = 1.$

11.3. $y = \frac{1}{1+x^2}, \quad x_0 = 2.$

11.4. $y = \frac{\sin x}{2-x}, \quad x_0 = 0.$

11.5. $y = \frac{2x}{1+x^2}, \quad x_0 = \sqrt{2}.$

11.6. $y = \frac{3x+1}{2x-1}, \quad x_0 = -1.$

11.7. $y = \sqrt{5-x^2}, \quad x_0 = 1.$

11.8. $y = \sqrt{4+2x-x^2}, \quad x_0 = 3.$

11.9. $y = \arctg 2x, \quad x_0 = 0.$

11.10. $y = \frac{x^2+1}{x-3}, \quad x_0 = 4.$

11.11. $y = \cos 2x - 2 \sin x, \quad x_0 = \pi.$

11.12. $y = \cos\left(2x - \frac{\pi}{2}\right) + 2, \quad x_0 = \frac{\pi}{2}.$

11.13. $y = \sqrt[3]{x-1}$, $x_0 = 2$.

11.14. $y = \ln x$, $x_0 = 1$.

11.15. $y = \frac{1}{x} \operatorname{arcctg} x$, $x_0 = 1$.

11.16. $y = 3^x + 3^{-2x}$, $x_0 = 1$.

11.17. $y = \operatorname{tg} 2x$, $x_0 = 0$.

11.18. $y = \sin(x+\pi) + 1$, $x_0 = \frac{\pi}{4}$.

11.19. $y = x^2 \ln x$, $x_0 = 1$.

11.20. $y = \cos^2 x$, $x_0 = \frac{\pi}{4}$.

11.21. $y = x^2 e^{-x}$, $x_0 = 1$.

11.22. $y = \sqrt{x+1}$, $x_0 = 4$.

11.23. $y = \ln(2e-x)$, $x_0 = e$.

11.24. $y = \frac{1}{3x+2}$, $x_0 = 2$.

11.25. $y = \cos x - \frac{2}{\pi} x^2$, $x_0 = \frac{\pi}{2}$.

11.26. $y = \sqrt[3]{x^2} - 20$, $x_0 = -8$.

11.27. $y = \frac{\sqrt{x}}{x+4}$, $x_0 = 4$.

11.28. $y = 8\sqrt[4]{x} - 70$, $x_0 = 16$.

11.29. $y = (x^2 - 5x + 7)e^x$, $x_0 = 0$.

11.30. $y = 2x + \frac{1}{x}$, $x_0 = 1$.

Задание 12. Составить уравнения касательных к графику функции $y = \frac{ax+b}{cx+d}$, параллельных или перпендикулярных к прямой $Ax + By + C = 0$.

12.1. $y = \frac{x+1}{x-1}$, параллельно прямой $2x + y - 2 = 0$.

12.2. $y = \frac{x-3}{x+1}$, параллельно прямой $x - y + 3 = 0$.

12.3. $y = \frac{x-3}{x+1}$, параллельно прямой $4x - y + 3 = 0$.

12.4. $y = \frac{-x+3}{x+1}$, параллельно прямой $x + y + 4 = 0$.

12.5. $y = \frac{-x+3}{x+1}$, параллельно прямой $4x + y - 3 = 0$.

12.6. $y = \frac{3x+2}{2x+2}$, параллельно прямой $2x - y + 4 = 0$.

12.7. $y = \frac{-3x+2}{-2x+4}$, параллельно прямой $2x + y - 4 = 0$.

12.8. $y = \frac{5x-1}{3x+3}$, параллельно прямой $2x - y + 7 = 0$.

12.9. $y = \frac{5x+3}{x+6}$, параллельно прямой $6x - 2y + 3 = 0$.

12.10. $y = \frac{5x+3}{-x-6}$, параллельно прямой $6x + 2y - 4 = 0$.

12.11. $y = \frac{4x+1}{4x+5}$, параллельно прямой $8x - 2y + 5 = 0$.

12.12. $y = \frac{4x-1}{4x-5}$, параллельно прямой $8x + 2y - 5 = 0$.

12.13. $y = \frac{6x-5}{3x+5}$, параллельно прямой $10x - 2y + 15 = 0$.

- 12.14.** $y = \frac{7x+3}{2x-2}$, параллельно прямой $10x + 2y - 13 = 0$.
- 12.15.** $y = \frac{9x-3}{x+1}$, параллельно прямой $3x - y + 7 = 0$.
- 12.16.** $y = \frac{x-8}{x+4}$, перпендикулярно к прямой $6x + 2y - 7 = 0$.
- 12.17.** $y = \frac{4x+6}{4x+3}$, перпендикулярно к прямой $6x - 2y + 7 = 0$.
- 12.18.** $y = \frac{3x+1}{x+3}$, перпендикулярно к прямой $4x + 2y - 9 = 0$.
- 12.19.** $y = \frac{-5x+1}{-2x+4}$, перпендикулярно к прямой $4x - 2y + 9 = 0$.
- 12.20.** $y = \frac{7x+1}{2x-2}$, перпендикулярно к прямой $3x - 3y + 1 = 0$.
- 12.21.** $y = \frac{8x+4}{6x+1}$, перпендикулярно к прямой $-4x + y - 7 = 0$.
- 12.22.** $y = \frac{8x-4}{6x-1}$, перпендикулярно к прямой $8x + 2y - 11 = 0$.
- 12.23.** $y = \frac{10x+7}{5x-9}$, перпендикулярно к прямой $10x - 2y + 21 = 0$.
- 12.24.** $y = \frac{10x+7}{-5x+9}$, перпендикулярно к прямой $5x + y - 10,5 = 0$.
- 12.25.** $y = -\frac{3x+3}{x+7}$, перпендикулярно к прямой $4x - 2y + 19 = 0$.
- 12.26.** $y = \frac{3x-1}{x-2}$, перпендикулярно к прямой $10x - 2y + 9 = 0$.
- 12.27.** $y = \frac{6x-1}{2x+5}$, перпендикулярно прямой $4x + 2y - 15 = 0$.
- 12.28.** $y = \frac{2x+2}{2x-1}$, перпендикулярно к прямой $12x - 2y + 21 = 0$.

12.29. $y = \frac{2x+2}{-2x+1}$, перпендикулярно к прямой $12x+2y-21=0$.

12.30. $y = \frac{3x-1}{x+2}$, перпендикулярно к прямой $14x+2y-25=0$.

Задание 13. Тело движется прямолинейно по закону $S(t)$. Найти скорость и ускорение тела в момент времени t_0 .

13.1. $S(t) = t^2 + 8\sqrt{(t+3)^3} - 2t + 1$, $t_0 = 1$.

13.2. $S(t) = 2t^2 + 4\sqrt{(t+2)^3} + 3t + 4$, $t_0 = 2$.

13.3. $S(t) = 5t^2 - \sqrt{(t-3)^3} + t - 1$, $t_0 = 4$.

13.4. $S(t) = 4t^2 + \sqrt{(t+3)^3} - 2t + 7$, $t_0 = 0$.

13.5. $S(t) = t^3 + \sqrt[3]{(t-1)^4} - \frac{3}{2}t^2 + 2$, $t_0 = 2$.

13.6. $S(t) = 3t^2 + 3\sqrt[3]{(t+2)^4} + 4t - 3$, $t_0 = 6$.

13.7. $S(t) = 2t^3 - 2\sqrt{(t-3)^3} - \frac{t^2}{2} - 5$, $t_0 = 4$.

13.8. $S(t) = \frac{t^3}{3} + 6\sqrt[3]{(t+7)^4} - t^2 - 3$, $t_0 = 1$.

13.9. $S(t) = \frac{2}{3}t^3 + \sqrt{(t-1)^3} - 2t^2 + 2t - 3$, $t_0 = 2$.

13.10. $S(t) = 7t^2 - 4\sqrt{(t+5)^3} - 3t + 10$, $t_0 = 4$.

13.11. $S(t) = \frac{t^3}{6} - 3\sqrt{(t+3)^3} + 8t + 10$, $t_0 = 4$.

13.12. $S(t) = 7t^2 - 9\sqrt[3]{(t-1)^4} - 5t - 3$, $t_0 = 2$.

13.13. $S(t) = \frac{5}{6}t^3 + 4\sqrt{(t-3)^3} - 8t - 7$, $t_0 = 4$.

- 13.14.** $S(t) = \frac{3}{2}t^2 + 5\sqrt{(t+1)^3} - 9t - 5,$ $t_0 = 3.$
- 13.15.** $S(t) = 4t^2 - 9\sqrt[3]{(t+3)^4} + 9t + 1,$ $t_0 = 5.$
- 13.16.** $S(t) = \frac{2}{3}t^2 - 3\sqrt[3]{(t-4)^4} - 2t^2 - t - 4,$ $t_0 = 5.$
- 13.17.** $S(t) = 5t^2 - 3\sqrt{(t+4)^3} - 4t - 1,$ $t_0 = 5.$
- 13.18.** $S(t) = 4t^2 + 5\sqrt{(t-6)^3} - 80t + 3,$ $t_0 = 7.$
- 13.19.** $S(t) = 10t^2 - 3\sqrt[3]{(t+1)^4} + 7t + 7,$ $t_0 = 0.$
- 13.20.** $S(t) = 2t^2 - 10\sqrt{(t-2)^3} + 90t - 4,$ $t_0 = 6.$
- 13.21.** $S(t) = 5t^2 + 3\sqrt[3]{(t-1)^4} - 20t + 1,$ $t_0 = 9.$
- 13.22.** $S(t) = t^3 - 6\sqrt[3]{(t+6)^4} + 93t - 4,$ $t_0 = 2.$
- 13.23.** $S(t) = t^2 + 8\sqrt{(t+3)^3} - 2t + 1,$ $t_0 = 7.$
- 13.24.** $S(t) = \frac{7}{18}t^3 - \sqrt[3]{(t-2)^4} - 2t^2 + 10,$ $t_0 = 3.$
- 13.25.** $S(t) = \frac{5}{9}t^3 - 2\sqrt[3]{(t+25)^4} + 12t^2 + 200,$ $t_0 = 1.$
- 13.26.** $S(t) = 6t^2 - 3\sqrt{(t+2)^3} + 4t - 3,$ $t_0 = 2.$
- 13.27.** $S(t) = 3t^2 - 6\sqrt{(t+1)^3} + 7t + 9,$ $t_0 = 0.$
- 13.28.** $S(t) = \frac{t^3}{3} + 3\sqrt[3]{(t+1)} - 2t^2 + t - 1,$ $t_0 = 0.$
- 13.29.** $S(t) = \frac{t^2}{4} + \sqrt{(t-2)^3} + 2t - 7,$ $t_0 = 11.$
- 13.30.** $S(t) = \frac{t^3}{6} + 3\sqrt[3]{(t+8)^4} + \frac{t^2}{2} - 38,$ $t_0 = 0.$

Задание 14. Найти приращение Δy и дифференциал dy функции $y = f(x)$ при переходе от точки x_0 к x .

14.1. $y(x) = 3x^2 - 4x + 5$, $x_0 = 1$, $x = 1,2$.

14.2. $y(x) = 4x^2 - 3x + 2$, $x_0 = 1$, $x = 1,3$.

14.3. $y(x) = 3x^2 - x$, $x_0 = 1$, $x = 1,2$.

14.4. $y(x) = 2x^2 + 5x - 1$, $x_0 = 3$, $x = 3,4$.

14.5. $y(x) = x^3 - 7x + 3$, $x_0 = 0$, $x = 0,3$.

14.6. $y(x) = (x+9)^3$, $x_0 = 2$, $x = 2,3$.

14.7. $y(x) = \sqrt{x}$, $x_0 = 0,25$, $x = 0,251$.

14.8. $y(x) = 2x^3 + x^2 + 3x$, $x_0 = 1$, $x = 1,2$.

14.9. $y(x) = \frac{x}{1+x^2}$, $x_0 = 1$, $x = 1,3$.

14.10. $y(x) = x^4 - x + 8$, $x_0 = 0$, $x = 0,4$.

14.11. $y(x) = (x+3)(x^2 - 2)$, $x_0 = 3$, $x = 3,5$.

14.12. $y(x) = 10x + 0,3$, $x_0 = 1$, $x = 1,2$.

14.13. $y(x) = x^2 - 5x - 7$, $x_0 = -2$, $x = -2,2$.

14.14. $y(x) = \frac{1}{4}x^4 + x^2 - 1$, $x_0 = 1$, $x = 1,6$.

14.15. $y(x) = (x^2 - 3)^4$, $x_0 = 0$, $x = 0,2$.

14.16. $y(x) = 5x^2 + 7x$, $x_0 = 5$, $x = 5,9$.

14.17. $y(x) = x^3 - x^2 + 1$, $x_0 = 1$, $x = 1,2$.

14.18. $y(x) = x^3 + x^2 + x$, $x_0 = 3$, $x = 3,6$.

- 14.19.** $y(x) = 9x^4 - 7x^2 + 9$, $x_0 = 1$, $x = 1,2$.
- 14.20.** $y(x) = 3x^3 - 7x + 1$, $x_0 = 0$, $x = 0,2$.
- 14.21.** $y(x) = 5x^3 - 2$, $x_0 = 5$, $x = 5,7$.
- 14.22.** $y(x) = (x^2 + 2)^2$, $x_0 = 1$, $x = 1,2$.
- 14.23.** $y(x) = x^2 - 13x$, $x_0 = 3$, $x = 3,5$.
- 14.24.** $y(x) = \frac{x}{x^2 - 2}$, $x_0 = 1$, $x = 1,2$.
- 14.25.** $y(x) = x^3 + 12x^2$, $x_0 = 0$, $x = 0,2$.
- 14.26.** $y(x) = x^5$, $x_0 = 7$, $x = 7,12$.
- 14.27.** $y(x) = x^5 + 2x - 8$, $x_0 = 2$, $x = 2,4$.
- 14.28.** $y(x) = x^2 - 4x$, $x_0 = 1$, $x = 1,2$.
- 14.29.** $y(x) = 9x^3 + \log_3 27$, $x_0 = 5$, $x = 5,6$.
- 14.30.** $y(x) = 5x - 4x^3$, $x_0 = 1$, $x = 1,8$.

Задание 15. Вычислить приближенно (с помощью дифференциала).

- 15.1.** $\sqrt[3]{8,06}$.
- 15.2.** $\sqrt[4]{16,02}$.
- 15.3.** $(1,021)^{11}$.
- 15.4.** $\frac{1}{\sqrt{4,016}}$.
- 15.5.** $\sqrt{0,98^3}$.
- 15.16.** $\frac{1}{\sqrt{9,02}}$.
- 15.17.** $\sqrt{(1,02)^3}$.
- 15.18.** $(3,02)^5$.
- 15.19.** $\arcsin(-0,04)$.
- 15.20.** $\sqrt[4]{81,01}$.

15.6. $\sqrt[3]{(1,03)^2}$.

15.7. $(2,997)^5$.

15.8. $\arcsin 0,08$.

15.9. $\sqrt[4]{17}$.

15.10. $\operatorname{arctg} 0,98$.

15.11. $\ln 1,01$.

15.12. $\sqrt[5]{31}$.

15.13. $\sqrt[3]{7,98}$.

15.14. $\sqrt[4]{81,02}$.

15.15. $(0,99)^{12}$.

15.21. $\operatorname{arctg} 1,02$.

15.22. $\ln 0,99$.

15.23. $\sqrt[5]{242}$.

15.24. $\sqrt[3]{27,01}$.

15.25. $\sqrt[4]{15,99}$.

15.26. $(0,98)^{10}$.

15.27. $e^{0,2}$.

15.28. $\sqrt{1,01^3}$.

15.29. $\sqrt{120}$.

15.30. $e^{-0,3}$.

Задание 16. Записать многочлен Тейлора 3-й степени для данной функции $y = f(x)$ в окрестности точки x_0 .

16.1. $y = \sin^2 x$, $x_0 = 0$.

16.16. $y = \cos^2 x$, $x_0 = 0$.

16.2. $y = \ln x$, $x_0 = 1$.

16.17. $y = \ln 2x$, $x_0 = 1$.

16.3. $y = \log_5 x$, $x_0 = 5$.

16.18. $y = e^x$, $x_0 = 1$.

16.4. $y = e^{x^2}$, $x_0 = 0$.

16.19. $y = e^{-x}$, $x_0 = 0$.

16.5. $y = e^{x^2}$, $x_0 = 1$.

16.20. $y = x^4$, $x_0 = 1$.

16.6. $y = \sin x$, $x_0 = \frac{\pi}{2}$.

16.21. $y = \cos x$, $x_0 = \frac{\pi}{2}$.

16.7. $y = \sin 3x$, $x_0 = 0$.

16.8. $y = \sin 2x$, $x_0 = \frac{\pi}{2}$.

16.9. $y = e^{\sin x}$, $x_0 = 0$.

16.10. $y = e^{3x}$, $x_0 = 1$.

16.11. $y = x^5$, $x_0 = 1$.

16.12. $y = \sin 2x$, $x_0 = \frac{\pi}{2}$.

16.13. $y = \sin^2 x$, $x_0 = 1$.

16.14. $y = \log_5 x$, $x_0 = 1$.

16.15. $y = \cos 3x$, $x_0 = \frac{\pi}{2}$.

16.22. $y = \cos 5x$, $x_0 = 0$.

16.23. $y = \cos 3x$, $x_0 = 0$.

16.24. $y = e^{\cos x}$, $x_0 = 0$.

16.25. $y = e^{x^2}$, $x_0 = 1$.

16.26. $y = \frac{1}{x}$, $x_0 = 1$.

16.27. $y = \cos^2 x$, $x_0 = 1$.

16.28. $y = e^{2x}$, $x_0 = 1$.

16.29. $y = e^x$, $x_0 = 1$.

16.30. $y = \ln x$, $x_0 = 2$.

Задание 17. Вычислить предел, используя правило Лопитала.

17.1. $\lim_{x \rightarrow 0} \frac{\sin^2 3x}{x^2}$.

17.2. $\lim_{x \rightarrow 1} \frac{x-1}{\ln x}$.

17.3. $\lim_{x \rightarrow 0} \frac{1-\cos x}{x^2}$.

17.16. $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x}$.

17.17. $\lim_{x \rightarrow 0} \frac{1 - \cos ax}{1 - \cos bx}$.

17.18. $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$.

$$17.4. \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x - \sin x}.$$

$$17.5. \lim_{x \rightarrow 0} \frac{\ln x}{\operatorname{ctg} x}.$$

$$17.6. \lim_{x \rightarrow 0+0} x^x.$$

$$17.7. \lim_{x \rightarrow 0} (\sin x)^{\operatorname{tg} x}.$$

$$17.8. \lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx}}{\sin x}.$$

$$17.9. \lim_{x \rightarrow 0} \frac{x - \operatorname{arctg} x}{x^3}.$$

$$17.10. \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\cos 3x}.$$

$$17.11. \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \operatorname{tg} x}{\cos 2x}.$$

$$17.12. \lim_{x \rightarrow 1} \frac{\ln x}{1 - x^3}.$$

$$17.13. \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x.$$

$$17.19. \lim_{x \rightarrow \infty} \frac{\ln x}{x}.$$

$$17.20. \lim_{x \rightarrow \pi} (\pi - x) \operatorname{tg} \frac{x}{2}.$$

$$17.21. \lim_{x \rightarrow 0} x \ln x.$$

$$17.22. \lim_{x \rightarrow \frac{\pi}{6}} \frac{1 - 2 \sin x}{\cos 3x}.$$

$$17.23. \lim_{x \rightarrow \frac{\pi}{2a}} \frac{1 - \sin ax}{(2ax - \pi)^2}.$$

$$17.24. \lim_{x \rightarrow 0} \frac{a^x - e^x}{\operatorname{tg} x}.$$

$$17.25. \lim_{x \rightarrow 1} \frac{\ln x}{1 - x^3}.$$

$$17.26. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\ln(1 + 2x)}.$$

$$17.27. \lim_{x \rightarrow 1} x^{\frac{1}{1-x}}.$$

$$17.28. \lim_{x \rightarrow 0} (\sin x)^{\operatorname{tg} x}.$$

$$\mathbf{17.14.} \lim_{x \rightarrow 0} \left(e^{2x} + x \right)^{\frac{1}{x}}.$$

$$\mathbf{17.15.} \lim_{x \rightarrow 0} (1 - e^{2x}) \operatorname{ctg} x.$$

$$\mathbf{17.29.} \lim_{x \rightarrow 0} \left(\frac{1}{x \sin x} - \frac{1}{x^2} \right).$$

$$\mathbf{17.30.} \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\ln(1 + 2x)}.$$

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Задание 18. Найти наибольшее и наименьшее значения функции $y = f(x)$ на отрезках.

18.1. $y(x) = \frac{1}{3}x^3 - 4x^2 + 12x + 1$, $[0;3]$, $[-1;1]$.

18.2. $y(x) = \frac{1}{4}x^4 + \frac{4}{3}x^3 - \frac{9}{2}x^2 - 36x$, $[-5;8]$, $[-1;4]$.

18.3. $y(x) = \frac{1}{3}x^3 - x^2 - 3x - 2$, $[-3;5]$, $[-4;0]$.

18.4. $y(x) = \frac{2}{4}x^4 + \frac{4}{3}x^3 - 16x^2 - 64x$, $[-5;5]$, $[-6;-1]$

18.5. $y(x) = 2x^3 - 8x^2 - 6x + 4$, $[-2;5]$, $[1;4]$.

18.6. $y(x) = \frac{49}{2}x^2 - \frac{8}{3}x^3 - 45x - 3$, $[0;6]$, $[-2;2]$.

18.7. $y(x) = \frac{1}{3}x^3 + \frac{7}{2}x^2 + 6x - 13$, $[-2;0]$, $[-7;0]$.

18.8. $y(x) = 7x^2 - \frac{1}{3}x^3 - 49x + 4$, $[-3;10]$, $[8;9]$.

18.9. $y(x) = \frac{1}{4}x^4 + \frac{13}{3}x^3 - \frac{1}{2}x^2 - 13x$, $[-15;2]$, $[-2;3]$.

18.10. $y(x) = \frac{31}{2}x^2 - x^3 + 22x - 7$, $[-1;12]$, $[2;13]$.

18.11. $y(x) = 2x^3 + 14x^2 + 32x - 3$, $[-5;0]$, $[-2.5;1]$.

18.12. $y(x) = \frac{25}{2}x^2 - \frac{1}{4}x^4 - \frac{4}{3}x^3 + 100x$, $[-7;7]$, $[3;6]$.

18.13. $y(x) = \frac{1}{3}x^3 + 5x^2 - 24x + 6$, $[-13;1]$, $[-15;7]$.

18.14. $y(x) = 7x^3 + \frac{5}{2}x^2 - 4x + 11$, $[-2;0]$, $\left[\frac{1}{5};3\right]$.

18.15. $y(x) = \frac{1}{3}x^3 + 3x^2 - 27x + 6$, $[-15;5]$, $[0;6]$.

18.16. $y(x) = 18x^2 - 396x - \frac{1}{4}x^4 + \frac{11}{3}x^3$, $[-8;12]$, $[0;7]$.

18.17. $y(x) = 12x^3 + 57x^2 + 18x - 27$, $[-4;5]$, $[-4;-2]$.

18.18. $y(x) = \frac{4}{3}x^3 + 10x^2 + 24x + 1$, $[-4;1]$, $[-2,5;0]$.

18.19. $y(x) = \frac{35}{3}x^3 + \frac{69}{2}x^2 - 36x + 17$, $[-3;2]$, $[0;7]$.

18.20. $y(x) = 50x^2 + 200x - \frac{1}{4}x^4 - \frac{2}{3}x^3$, $[-12;12]$, $[0;15]$.

18.21. $y(x) = 6x^3 - \frac{87}{2}x^2 + 84x - 11$, $[-2;5]$, $[2;7]$.

18.22. $y(x) = \frac{153}{2}x^2 - 5x^3 - 162x + 21$, $[-3;10]$, $[5;11]$.

18.23. $y(x) = \frac{2}{3}x^3 + \frac{35}{2}x^2 + 143x - 13$, $[-13;0]$, $[-9;-4]$.

18.24. $y(x) = x^3 + 28x^2 - 220x + 13$, $[-25;5]$, $[0;7]$.

18.25. $y(x) = \frac{1}{4}x^4 + \frac{7}{3}x^3 - \frac{121}{2}x^2 - 847x + 21$, $[-15;21]$, $[9;12]$.

18.26. $y(x) = \frac{26}{3}x^3 + 43x^2 - 72x - 1$, $[-5;2]$, $[-10;0]$.

18.27. $y(x) = \frac{85}{3}x^3 - 32x^2 - 77x + 2$, $[-2;2]$, $[1;9]$.

18.28. $y(x) = \frac{3}{2}x^4 + x^3 - 192x^2 - 192x + 72$, $[-10;10]$, $[2;9]$.

18.29. $y(x) = \frac{1}{3}x^3 + \frac{15}{2}x^2 - 496x - 7$, $[-32;17]$, $[0;3]$.

18.30. $y(x) = 494x - \frac{1}{3}x^3 - \frac{7}{2}x^2 + 5$, $[-30;20]$, $[15;25]$.

Задание 19. Найти асимптоты и схематически построить график функции.

19.1. $y = \frac{x^3 - 3x^2 + 7}{x^4 - 4}$.

19.2. $y = \frac{x}{\sqrt{1 - x^2}}$.

19.3. $y = \sqrt{\frac{1-x}{x}}$.

19.4. $y = \frac{6(x^2 - 4)}{3x^2 + 8}$.

19.5. $y = \frac{\sqrt{1+x^2}}{x}$.

19.6. $y = \frac{x^2 - 2x}{x - 1}$.

19.7. $y = \frac{\sqrt{4x^4 + 1}}{|x|}$.

19.8. $y = x + \frac{1}{x^2}$.

19.9. $y = \frac{2x^4 + x^3 + 1}{x^3}$.

19.16. $y = \frac{4}{x^4 - 2x^2}$.

19.17. $y = \frac{(x+1)^3}{(x+2)^2}$.

19.18. $y = \sqrt{\frac{x}{x-2}}$.

19.19. $y = \frac{\sqrt{4+x^2}}{2x}$.

19.20. $y = \frac{\sqrt{9x^4 + 1}}{|x|}$.

19.21. $y = 2x - \frac{1}{x^2}$.

19.22. $y = \frac{4+x^2}{x^3 - 9x}$.

19.23. $y = \frac{x}{2x-1} + x$.

19.24. $y = \frac{2x^2 + 3x - 5}{x(x-4)}$.

19.10. $y = \frac{x^2 - 6x + 3}{x - 3}.$

19.11. $y = \frac{x^5}{x^4 - 1}.$

19.12. $y = \frac{x^2 + 1}{\sqrt{x^2 - 1}}.$

19.13. $y = \frac{x}{x^2 - 4x + 3}.$

19.14. $y = \frac{x^2}{\sqrt{x^2 - 1}}.$

19.15. $y = \frac{x^2 - 1}{x^2 - 5x + 6}.$

19.25. $y = \frac{x^2 + 5}{x^2 - 1} + 2x.$

19.26. $y = \frac{x^4}{(1+x)^2}.$

19.27. $y = \frac{1}{2x^2 + x - 1}.$

19.28. $y = \frac{x^2}{4 - x^2}.$

19.29. $y = \frac{2x^2 - 9}{\sqrt{x^2 - 1}}.$

19.30. $y = \frac{x^2 - 11}{4x - 3}.$

Задание 20. Провести полное исследование и построить график функции $y = f(x)$.

20.1. а) $y = \frac{x^2}{x - 2};$ б) $y = (x+2)e^{-x};$

в) $y = \sqrt[3]{x+1} - \sqrt[3]{x-1}.$

20.2. а) $y = \frac{x^2 - 4x + 1}{x - 4};$ б) $y = xe^x;$

в) $y = \sqrt[3]{x^2 - 2x}.$

20.3. а) $y = \frac{x^2 - 5x}{1 - x};$ б) $y = (x-2)e^{3-x};$

в) $y = \sqrt[3]{(x+1)^2} + \sqrt[3]{(x-1)^2}.$

20.4. а) $y = \frac{6 - 2x^2}{x - 2};$ б) $y = (x - 1)e^{2x};$

в) $y = \frac{1}{\sqrt[3]{x+1}} + \frac{1}{\sqrt[3]{x-1}}.$

20.5. а) $y = \frac{3x^2}{2-x};$ б) $y = (2-x)e^x;$

в) $y = \sqrt[3]{1-x^3}.$

20.6. а) $y = \frac{7x - x^2}{x - 3};$ б) $y = xe^{-2x};$

в) $y = \sqrt[3]{x+1} + \sqrt[3]{x-1}.$

20.7. а) $y = \frac{x^2}{3-x};$ б) $y = (x+1)e^{2x};$

в) $y = \sqrt[3]{x^3 + 1} + \sqrt[3]{x^3 - 1}.$

20.8. а) $y = \frac{x^2 + 16}{4x};$ б) $y = (3-x)e^{x-2};$

в) $y = \frac{x^3}{\sqrt{x^4 + 1}}.$

20.9. а) $y = \frac{x^2 - x + 1}{1-x};$ б) $y = (4-x)e^{x-3};$

в) $y = \frac{x}{\sqrt{x^2 + 1}}.$

20.10. а) $y = \frac{x^2}{x+2};$ б) $y = xe^{3x};$

в) $y = \frac{x^3}{\sqrt[3]{x^3 + 2}}.$

20.11. а) $y = \frac{2x^2}{x-1};$ б) $y = (x-3)e^{-x};$

в) $y = \frac{x^3}{\sqrt[3]{x^3 - 4}}.$

20.12. а) $y = \frac{3x^2}{x-2};$ б) $y = (x-5)e^{2x};$

в) $y = \frac{x^3}{\sqrt[3]{(x^3 + 2)^2}}.$

20.13. а) $y = \frac{x^2 - 3x + 3}{x-1};$ б) $y = xe^{-x};$

в) $y = \frac{x^2}{\sqrt{x^2 + 1}}.$

20.14. а) $y = \frac{x^2}{x+1};$ б) $y = (x+1)e^{-2x};$

в) $y = \frac{\sqrt[3]{x^3 + 2}}{x}.$

20.15. а) $y = \frac{3-x^2}{x+2};$ б) $y = (x+4)e^{2x};$

в) $y = \frac{x}{\sqrt{(x^3 + 1)^2}}.$

20.16. а) $y = \frac{x^2}{x-5};$

б) $y = (x+1)e^{-x};$

в) $y = \frac{x^2}{\sqrt{|x^2 - 1|}}.$

20.17. а) $y = \frac{x^2 - x + 1}{x - 1};$

б) $y = xe^{2x-1};$

в) $y = \frac{\sqrt{|x^2 - 3|}}{x}.$

20.18. а) $y = \frac{-x^2 - 4}{2x};$

б) $y = (x-1)e^{-x};$

в) $y = \sqrt[3]{|x^2 - 1|}.$

20.19. а) $y = \frac{x^2}{4+x};$

б) $y = (x-3)e^{x-2};$

в) $y = \sqrt{|x^2 - 2|^3}.$

20.20. а) $y = \frac{2x^2 - 6}{x - 2};$

б) $y = (x-4)e^{x-3};$

в) $y = \sqrt{8x^2 - x^4}.$

20.21. а) $y = \frac{x^2}{5-x};$

б) $y = xe^{2-x};$

в) $y = \frac{x-2}{\sqrt{x^2+1}}.$

20.22. а) $y = \frac{x^2 - 4x + 1}{4-x};$

б) $y = (x+2)e^{2x};$

в) $y = \sqrt[3]{x^3 - x^2 - x + 1}.$

20.23. а) $y = \frac{x^2}{x-3};$

б) $y = (x-4)e^x;$

в) $y = \sqrt{(x-1)(x-2)(x-3)}.$

20.24. а) $y = \frac{x^2 + 11}{x-5};$

б) $y = (x-4)e^{-x};$

в) $y = \sqrt[3]{x^2} - \sqrt[3]{x^2 + 1}.$

20.25. а) $y = \frac{x^2 + 4}{2x};$

б) $y = (x-1)e^x;$

в) $y = \sqrt{\frac{|1+x|^3}{x}}.$

20.26. а) $y = \frac{x^2 - 5x}{x-1};$

б) $y = (5-x)e^{2x};$

в) $y = \frac{x}{\sqrt[3]{x^2 - 1}}.$

20.27. а) $y = \frac{3x - x^2 - 3}{x - 1};$ б) $y = (x - 2)e^{2-x};$

в) $y = -\sqrt{8x^2 - x^4}.$

20.28. а) $y = \frac{x^2}{2-x};$ б) $y = (3-x)e^{-x};$

в) $y = \frac{x^2 \sqrt{x^2 - 1}}{2x^2 - 1}.$

20.29. а) $y = \frac{x^2 - 3}{x + 2};$ б) $y = (x - 4)e^{4-x};$

в) $y = 1 - x + \sqrt{\frac{x^3}{3+x}}.$

20.30. а) $y = \frac{2x^2}{1-x};$ б) $y = (x-1)e^{3x-1};$

в) $y = \sqrt[3]{\frac{x^2}{x+1}}.$